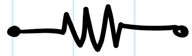
Filters and Bode Plots

Tuesday, June 30, 2020 8:22 PM

Passive Devices over Frequency


Resistor: $Z_R = R$ for all ω


Capacitor: $Z_C = \frac{1}{j\omega C}$

↳ small freq. = high impedance ← open

↳ high freq. = low impedance ← short

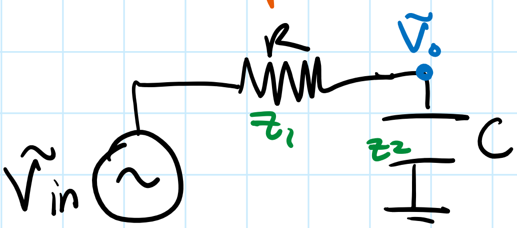

Inductor: $Z_L = j\omega L$

↳ small freq. = low impedance ← short

↳ high freq. = high impedance ← impedance

Low Pass Filter

Small ω passes through, high ω doesn't



If $\frac{1}{j\omega C}$ is large (small ω), then most of the voltage will be dropped here (after Vo).

If $\frac{1}{j\omega C}$ is small, the voltage will be dropped before Vo.

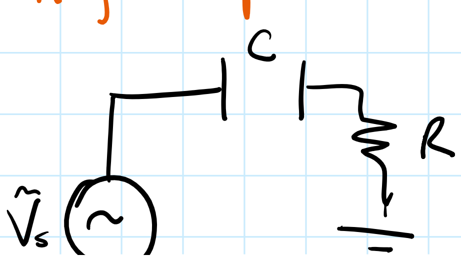
$Z_1 \gg Z_2$ for small freq.

$Z_1 \ll Z_2$ for high freq.

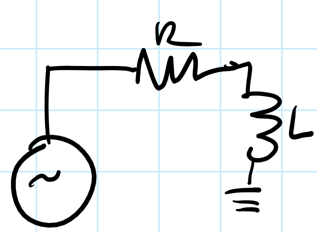
$H(j\omega)$ for RC: $\frac{Z_C}{Z_R + Z_C} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + 1} = \frac{1}{1 + \frac{j\omega}{1/RC}}$, $1/RC = \omega_p$

High Pass Filter

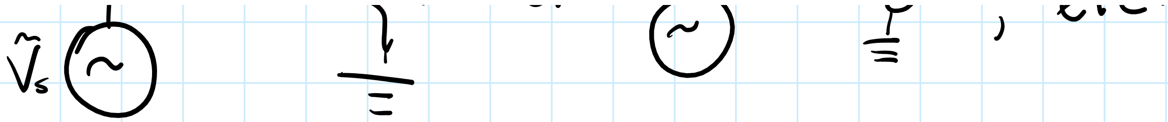
High ω passes through, small ω doesn't



or



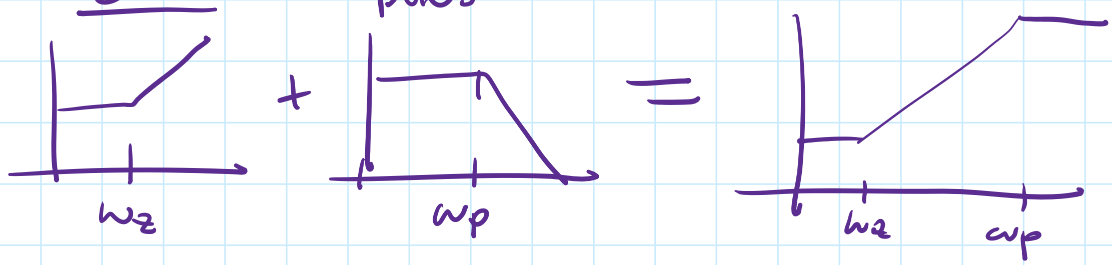
, etc.



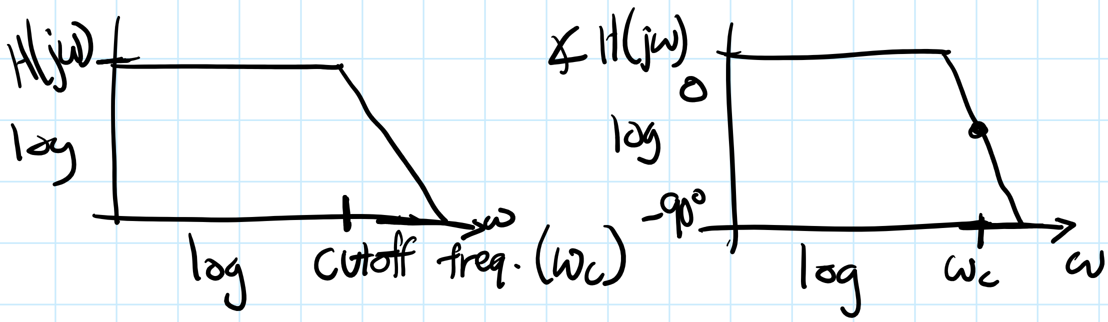
$Z_1 >> Z_2$ for small freq.
$Z_1 << Z_2$ for high freq.

Bode Plots

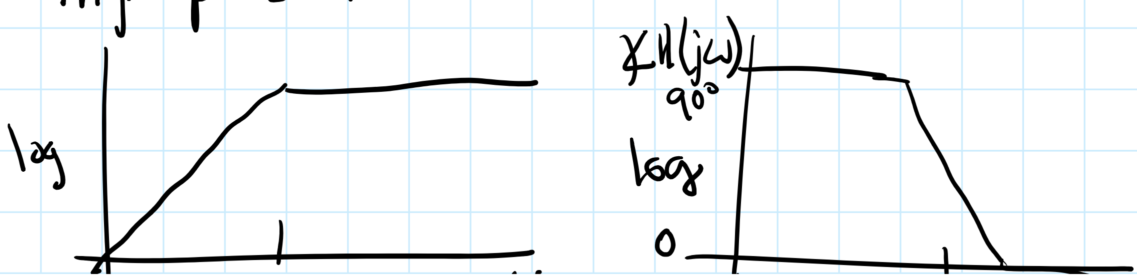
$$H(j\omega) = \underbrace{\frac{(j\omega)^n\left(1+\frac{j\omega}{\omega_{z1}}\right)\cdots}{(j\omega)^m\left(1+\frac{j\omega}{\omega_{p1}}\right)\cdots}}_{\text{factored transfer function}} \begin{array}{l} \leftarrow \text{zeros} \\ \leftarrow \text{poles} \end{array}$$

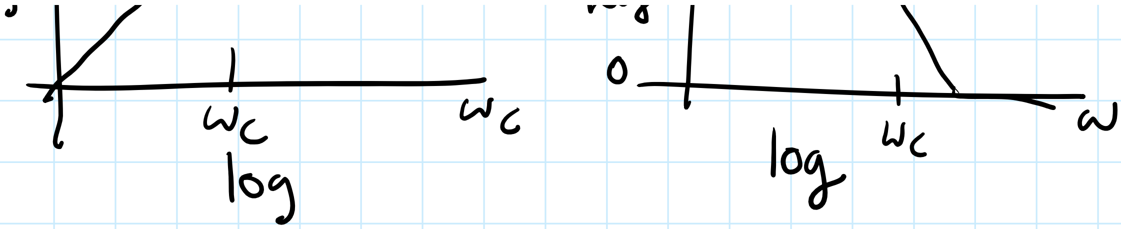


Low pass filter.



High pass filter:

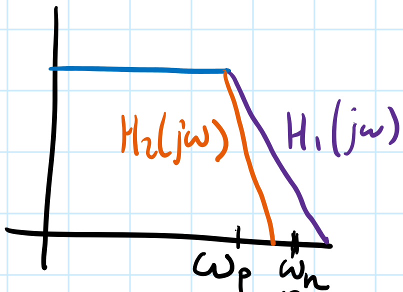




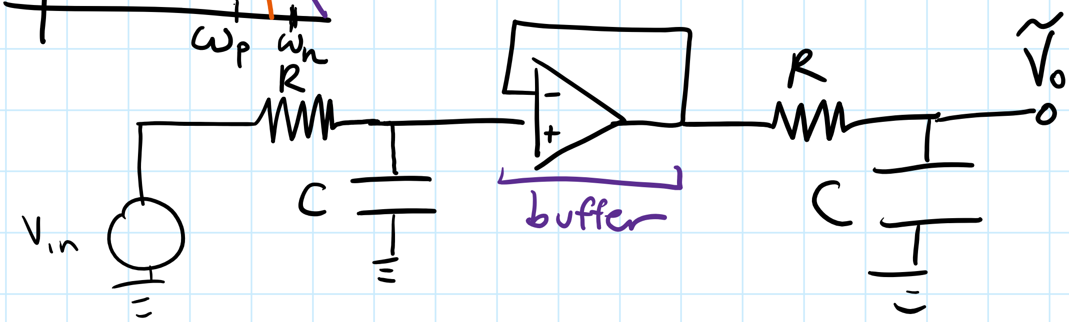
Second Order Filters

- Filter that has more than one pole/zero

Example: two poles at same value, different transfer functions

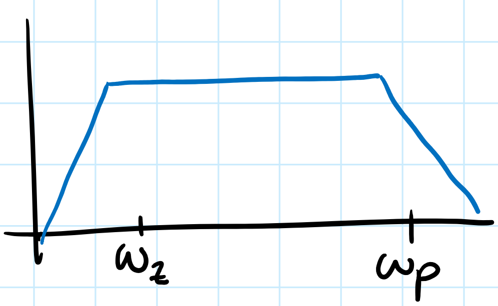


$$H(j\omega) = \frac{1}{\left(1+\frac{j\omega}{\omega_c}\right)\left(1+\frac{j\omega}{\omega_c}\right)}$$



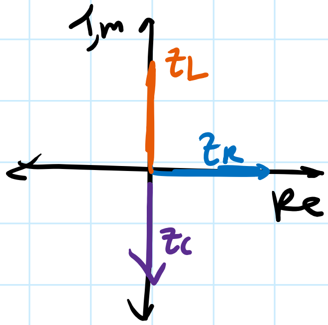
Significance:
- multiply j by j to get fully real result
- make band pass filters
- filter out noise that is very similar to actual signal

Band Pass Filter



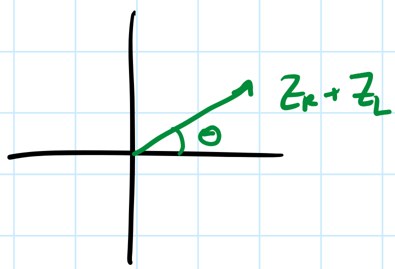
$$H(\omega j) = \frac{\frac{j\omega}{\omega_z}}{\left(1+\frac{j\omega}{\omega_p}\right)\left(1+\frac{j\omega}{\omega_z}\right)}$$

→ Resonance



$Z_R = R$
$Z_L = j\omega L$
$Z_C = \frac{1}{j\omega C}$

★ If we take these components in series, the phase will change...



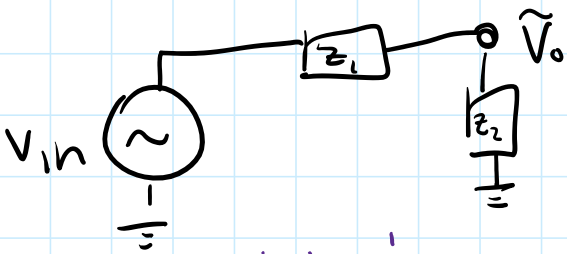
★ What if we combine all three?

$$Z_R + Z_L + Z_C = R + j\omega L + \frac{-1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

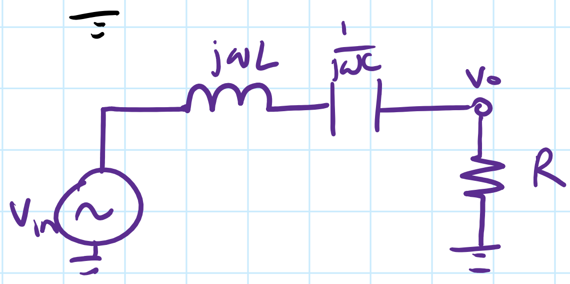
Resonant Frequency: when $\omega L - \frac{1}{\omega C} = 0$

⇓

Can be used to make a resonant band pass filter.

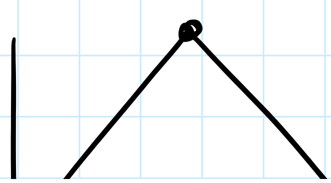


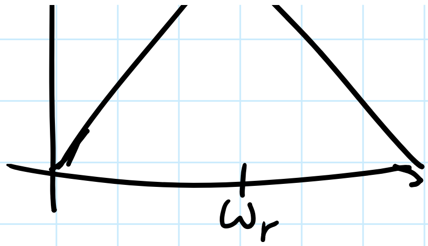
$$\begin{cases} H(j\omega) = \frac{Z_2}{Z_2 + Z_1} \\ Z_1 = 0 \text{ if } \omega = \omega_r \end{cases}$$



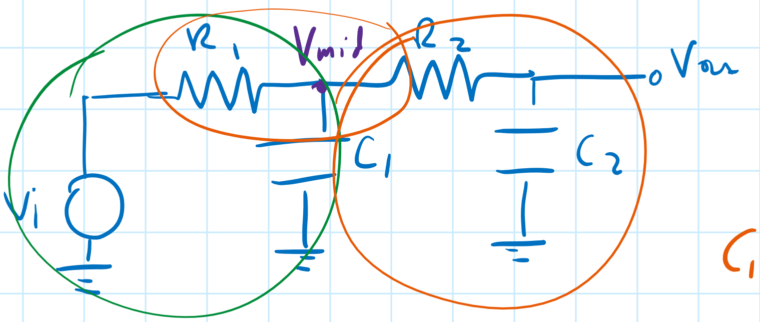
$$H(j\omega) = \frac{R}{R + \left(j\omega L + \frac{1}{j\omega C}\right)}$$

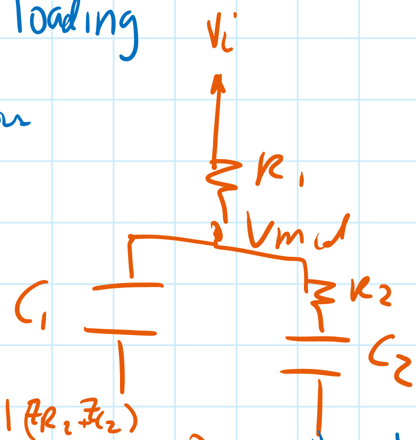
$$\lim_{\omega \to -\infty} H(j\omega) = \lim_{\omega \to \infty} H(j\omega) = 0$$





Ex: second order filter without loading





$$H_1(\omega) = \frac{Z_{C_1}}{Z_{R_1}+Z_{C_1}}$$

$$H_2(\omega) = \frac{Z_{C_1} \,||\, (Z_{R_2}+Z_{C_2})}{Z_{R_1} + (Z_{C_1} \,||\, (Z_{R_2}+Z_{C_2}))}$$ ← voltage divider of C_1 and $(R_2 + C_2)$ in parallel

$$= \frac{Z_{R_2}Z_{C_1} + Z_{C_1}Z_{C_2}}{Z_{R_1}Z_{C_1} + Z_{R_1}Z_{R_2} + Z_{R_1}Z_{C_2} + Z_{R_2}Z_{C_1} + Z_{C_1}Z_{C_2}}$$

$$= \frac{1 + j\omega R_2 C_2}{(1-\omega^2 R_1 R_2 C_1 C_2) + j\omega(R_1C_1 + R_2C_2 + R_1C_2)}$$

=