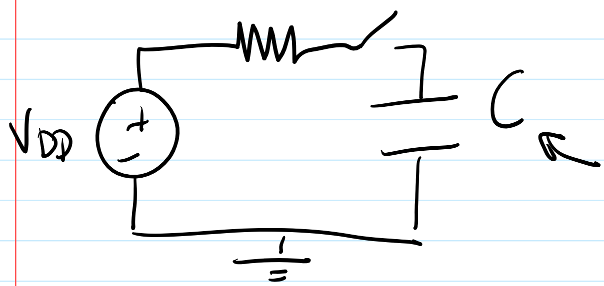
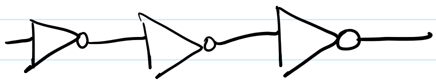
RC Circuits

Wednesday, June 24, 2020 11:23 AM



* Why do we care about charge delay?

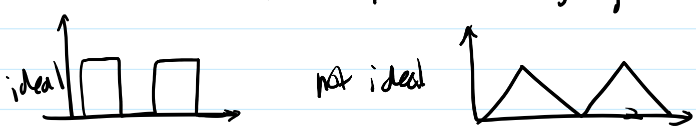


very common buffer design

each of these inverters has transistors. each transistor has capacitors.

→ Want to speed up clock speed → capacitor needs to charge faster

If we set the clock speed, charging will interfere w/ signal:



(delay can be useful for making clocks. larger cap = longer charge time)
- touchscreens

The Equations:

$I_R = I_C$

$$C\frac{dV(t)}{dt} = \frac{V_R(t)}{R}, \quad V_R(t) = V_{DD} - V_c(t)$$

$$\Downarrow$$

$$\frac{dV_c(t)}{dt} = \frac{(V_{DD} - V_c(t))}{RC}$$

$$\frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = \frac{V_{DD}}{RC}$$

Differential Equations

$$\frac{d}{dt}x(t) + ax(t) = b$$

Use the Solution Principle to get

$$x(t) = \underbrace{x_h(t)}_{\text{homogenous}} + \underbrace{x_p(t)}_{\text{particular}}$$

↳ sol. to $\frac{d}{dt}x(t) + ax(t) = 0$

$\Downarrow$

↳ sol. to $\frac{d}{dt}x(t) + ax(t) = 0$

☆ eigenfunction! $\frac{d}{dt}x(t) = -ax(t)$

↳ $x(t) = C_0 e^{-at}$

Initial condition: $x(0) = x_0 = C_0 e^0 = C_0$

$$\frac{d}{dt}x(t) = x_0 e^{-at}$$

Uniqueness Thm. of Differential Equations

If a function solves a diff eq then it is correct and unique,

1) an initial condition $x(t_0)$ exists

2) the solution of $x(t)$ is continuous over the desired output range and includes $x(t_0)$

Solve $\frac{d}{dt}x(t) + ax(t) = b$.

$$\frac{d}{dt}x(t) = b - ax(t)$$

$$\int \frac{\frac{d}{dt}x(t)}{x(t) - b/a} = \int -a\,dt$$

$$\ln\left|x(t) - b/a\right| = -at + C$$

$$x(t) = b/a + C'e^{-at}$$

$$x(t) = b/a\,(x_0 - b/a)\,e^{-at}$$

↳ plug in constants from RC equation to get

$$V_o(t) = \frac{V_{DD}/RC}{1/RC}\left(0 - \frac{V_{DD}/RC}{1/RC}\right) e^{\frac{-t}{RC}}$$

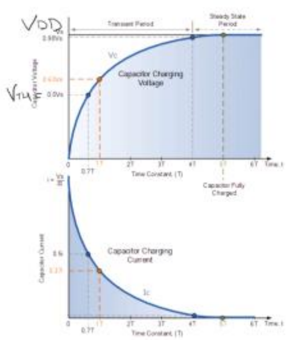
$V_o(t) = \frac{\quad}{1/RC}\left(0 - \frac{RC}{1/RC}\right) e^{RC}$

$a = \frac{1}{RC}$

$b = \frac{V_{DD}}{RC}$

$$V_o(t) = V_{DD}\left(1 - e^{-t/RC}\right)$$

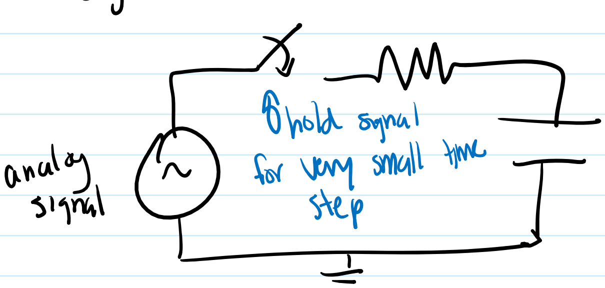
★ RC = time constant

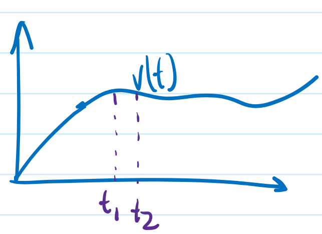


Time Constant	RC Value	Percentage of Maximum
		Voltage
0.5 time constant	0.5T = 0.5RC	39.3%
0.7 time constant	0.7T = 0.7RC	50.3%
1.0 time constant	1T = 1RC	63.2%
2.0 time constants	2T = 2RC	86.5%
3.0 time constants	3T = 3RC	95.0%
4.0 time constants	4T = 4RC	98.2%
5.0 time constants	5T = 5RC	99.3%

Application of time constant: ADC

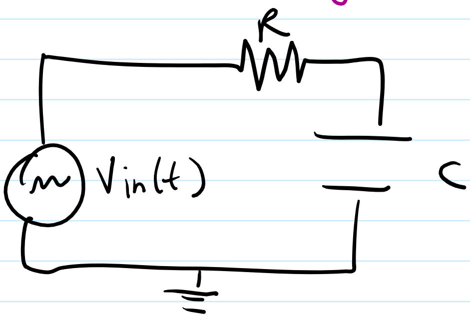
An Analog-to-digital converter samples an analog signal at time steps and converts them into digital signals





At t_1, switch closes
At t_2, switch opens and ADC takes voltage

Time Varying Inputs



$$\frac{d}{dt}V_o(t) + \frac{1}{RC}V_o(t) = \frac{V_{in}(t)}{RC}$$

Generalized problem: Solve

$$\frac{d}{dt}x(t) + a x(t) = g(t)$$

- Product rule: $\frac{d}{dt}x(t)y(t) = \frac{d}{dt}y(t)x(t) + \frac{d}{dx}x(t)y(t)$

$$= \frac{d}{dt}x(t) + a \quad x(t) = g(t)$$

$$= \frac{d}{dt}x(t)y(t) + a\,x(t)y(t) = g(t)y(t)$$

$$\frac{d}{dt}x(t)y(t) + \frac{d}{dt}\left(\int a\,x(t)y(t) = g(t)y(t)\right)$$

$$\star\; y(t) = e^{at}$$

Convert to Product Rule.

$$\int \frac{d}{dt}\left(x(t)y(t)\right) = \int g(t)\,y(t)\,dt$$

$$x(t)e^{at} = \int g(t)e^{at}\,dt$$

$$\boxed{x(t) = e^{-at}\int g(t)e^{at}\,dt}$$ homogenous solution

$(+Ce^{-at})$

⇓

in an RC circuit, $a = \frac{1}{RC}$, $x(t) = V_c(t)$, $g(t) = V_{in}(t)$

in an RC circuit, $a = \frac{1}{RC}$, $x(t) = V_c(t)$, $g(t) = V_{in}(t)$

$$V_c(t) = e^{-\frac{t}{RC}} \int_{t_0}^{t} \frac{V_{in}(\theta)}{RC} e^{\theta/RC} d\theta$$

Check using a constant: $V_{in}(t) = V_{dd}$, $V_o(0) = 0$

$$V_c(t) = e^{-t/RC} \int_{t_0}^{t} \frac{V_{dd}}{RC} e^{\theta/RC} d\theta + Ke^{-t/RC}$$

$$= e^{-t/RC} \cdot \frac{V_{dd}}{RC} e^{t/RC} \cdot RC + Ke^{-t/RC}$$

$$= V_{dd} + Ke^{-t/RC}$$

$$V_{dd} + Ke^{-\frac{0}{RC}} = 0$$

$$K = -V_{dd}$$

$$V_o(t) = V_{dd} - V_{dd} e^{-t/RC} = V_{dd}(1 - e^{-t/RC})$$

With an exponential $V_{in} = \tilde{V} e^{st}$:

$$V_o(t) = e^{-t/RC} \int \frac{\tilde{V}}{RC} e^{\theta(s + \frac{1}{RC})} + Ke^{-t/RC}$$

$$= e^{-t/RC} \frac{1}{(s + \frac{1}{RC})} \frac{\tilde{V}}{RC} e^{t(s + \frac{1}{RC})} + Ke^{-t/RC}$$

$$V_o(t) = \frac{\tilde{V}}{sRC + 1} e^{st} + Ke^{-t/RC}$$

$\Downarrow$

Very similar to voltage divider!

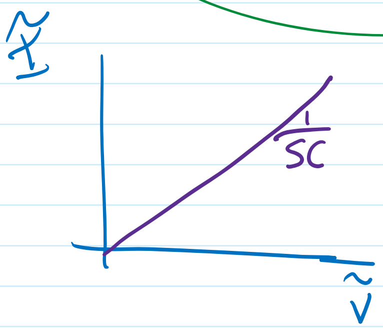
$$\frac{\tilde{V}}{sRC+1} = \tilde{V} \left(\frac{\frac{1}{sC}}{R + \frac{1}{sC}} \right)$$

with currents: $I_c = \frac{d}{dt} V_c \cdot C = \frac{d}{dt} \tilde{V} e^{st} \cdot C$

$$= s\tilde{V} e^{st} \cdot C$$

So $\frac{\tilde{V}}{\tilde{I}} = \frac{1}{sC} = R_c$

$\tilde{x}$.



example usage: multiple components

$Z_L = sL$

$Z_C = \frac{1}{sC}$

$Z_R = R$

voltage divider equivalences

$$V_o = V_{in} \frac{Z_C}{Z_C + Z_L}$$

$$= V_{in} \frac{\frac{1}{sC}}{\frac{1}{sC} + sL}$$

Steady State: wait until $e^{-t/RC}$ is negligible and $K \approx 0$ so that the exponential $Ke^{-t/RC}$ becomes insignificant