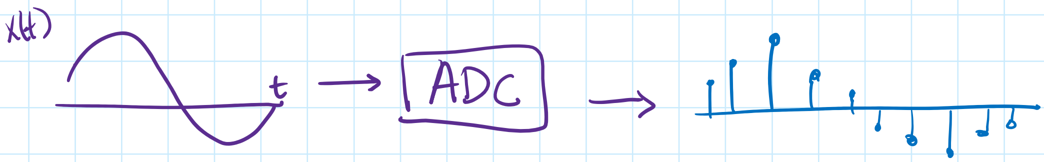
Sampling and Aliasing

Tuesday, August 11, 2020 11:22 AM

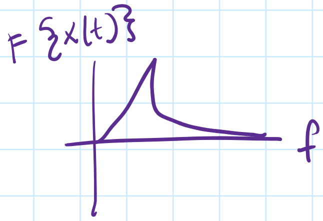
The world is continuous, but computers are discrete. How do we convert?

Sampling

Converting a continuous signal into discrete by reading the value at each time step



DFT to ⇓ Frequency domain



- find out how much frequency is in signal
- filter design
- signal processing
- project signal into orthogonal basis
  - ↳ phasor domain
  - ↳ convolutions

Sampling Frequency

Let $u_k = e^{j\frac{2\pi k n}{N}} = e^{j\omega}$ where $\omega = \frac{2\pi k}{N}$.

$\frac{k}{N} = \frac{f}{f_s}$ ← sampling frequency

Aliasing

Taking Fx of sinusoid and observing a repeated signal due to fast sampling frequency

- only occurs during sampling - not in continuous time

example: if $f = 40$ Hz, and we take 10 samples:

$\frac{k}{N} = \frac{40}{10} = 4, \; \underline{10 - 4}$

↳ conjugate appears due to aliasing

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Creating a signal where $X[n] = F\,x[n]$:

$$x(t) = \alpha_1 \cos(2\pi \cdot \underline{90\,\text{Hz}}\, t) - \alpha_2 \sin(2\pi \cdot 60\,\text{Hz}\, t)$$

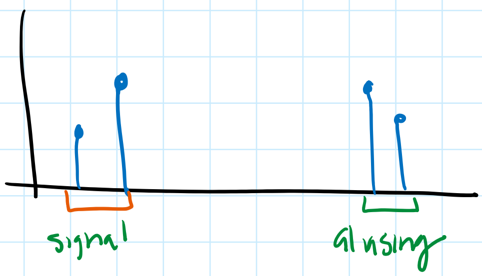
or $$x(t) = \alpha_1 \cos(2\pi \cdot \underline{110\,\text{Hz}}\, t) + \alpha_2 \sin(2\pi \cdot 140\,\text{Hz} \cdot t)$$

aliased frequencies

Sampling Theorem

Nyquist Sampling Theorem:

- Let all sampled frequencies be positive. All aliases are copies:



Theorem: A band-limited, continuous time signal can be perfectly reconstructed from its samples if the sampling frequency f_s is over twice as fast as the signal's highest frequency component.

- fast sampling → get high frequency details w/o aliasing
- long time period → low frequencies

↳ Ideal: fast f_s, long N

Realistic: $N = \frac{f_s}{f_{min}}$